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Technical Notes

Mistuning Identification of Blisks at Higher Frequencies

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I. Introduction

THE design of a single stage of a turbomachinery rotor, or bladed disk, typically features cyclic symmetry. However, there are always small structural deviations among blades, called mistuning, which can result in a dramatic increase in the maximum blade stress. From a modeling perspective, mistuning destroys the cyclic symmetry, such that the full-bladed disk needs to be modeled. Therefore, several efficient approaches for reduced-order modeling of mistuned bladed disks have been developed. For example, Lim et. al [1] and Feiner and Griffin [2] all presented reduced-order models (ROMs) for modeling mistuning.

In addition to modeling, the identification of mistuning is also very important. Thus, several mistuning identification methods have been developed that use experimental response data for the full blisk to extract individual blade mistuning parameters. These methods range from techniques based on lumped parameter models [3,4] to more advanced approaches requiring information from a finite element model [5–16]. A simplifying assumption is often made when modeling mistuning, namely, that mistuning is proportional. That means that the physical variations in each sector are considered to be proportional to the mass and/or stiffness matrices of the tuned sector. However, physical variability among sectors can be present in a nonuniform way in each sector. To address this, Lim [13] and Sinha et al. [16] present identification techniques that are flexible enough to identify the properties of blisks that are not restricted to proportional mistuning. These techniques are very different. The former [13] identifies variability in blade stiffness characterized by using variations in cantilevered blade frequencies. This method uses system forced response measurements as input. The latter [16] uses accurate measurements of the geometric variations among the blades as input in order to compute natural frequencies and mode shapes of each blade. In particular, Sinha et al. [16] presented results based on that technique for the first 19 blade-alone modes of a blisk (which reach higher frequency regions). The combination of work on modeling and identification of mistuning in blisks has made it possible to accurately model blisks at a fraction of the computational cost compared with finite element analysis. Most of the current techniques have been validated for predictions in lower-frequency ranges, and one [16] has been validated in the higher frequency ranges.

This work focuses on mistuning modeled as variations in the cantilevered blade frequencies and deals with how to use vibration measurements to identify the sector-to-sector variability in those frequencies. In particular, the work tackles the issue of selecting measurement points to use for identification, with particular attention being paid to the vibration of smaller regions of each blade, which takes place for higher modes. Note that, herein, mistuning is characterized by variations in cantilevered blade frequencies. Such a characterization of mistuning implies that a single physical mistuning pattern is characterized by multiple frequency-mistuning patterns. Each of the frequency-mistuning patterns corresponds to a given cantilevered blade mode. For example, in this work, we use a single physical mistuning pattern and characterize it by six frequency-mistuning patterns, corresponding to the first six cantilevered blade modes. These multiple frequency-mistuning patterns are identified (for a single physical mistuning pattern) by using the observation that each of the cantilevered blade modes dominates the dynamics in a different frequency range (i.e., for a different tuned mode family). In addition, the effective independence distribution vector (EIDV) [17,18] method is used in an iterative approach to identify measurement points that yield the highest accuracy for identification given a limited amount of measurement data to be collected. Demonstrating that the identification method can identify such multiple frequency-mistuning patterns ensures that the appropriate parameters can be identified and used to model the realistic case of nonuniform physical mistuning in each blade.

II. Theory

A. Component Mode Mistuning Overview

The ROM used for the identification procedure herein is based on component mode mistuning (CMM) [19]. This ROM is based only on tuned system modes as a basis and can be expressed as

$$\begin{aligned} -\omega^2\mathbf{p} + (1+j\gamma)[\mathbf{\Lambda}^s + \mathbf{\Lambda}^{\delta,s} + \mathbf{q}^T\mathbf{\Phi}^{cb^T}\mathbf{K}^\delta\mathbf{\Phi}^{cb}\mathbf{q}]\mathbf{p} = \mathbf{f}^s \\ -\omega^2\mathbf{p} + (1+j\gamma)[\mathbf{\Lambda}^s + \mathbf{\Lambda}^{\delta,s} + \mathbf{q}^T\mathbf{\Lambda}^{\delta,cb}\mathbf{q}]\mathbf{p} = \mathbf{f}^s \end{aligned} \tag{1}$$

where $\mathbf{p}$ is the modal amplitude coordinate (i.e., $\mathbf{p}$ contains the amplitudes of tuned system modes used as a basis), $\mathbf{\Lambda}^{\delta,cb}$ are the differences of the cantilevered blade eigenvalues between the tuned and mistuned models, $\mathbf{q}$ are the participation factors of the cantilevered blade modes in the blade portion of the tuned system modes, and $\mathbf{f}^s$ is the modal forcing. The diagonal matrix $\mathbf{\Lambda}^{\delta,s}$ contains the differences in the eigenvalues of the assumed nominal tuned system and the actual tuned system [13]. The variable $\mathbf{q}$ represents the participation the cantilevered blade mode shapes in the system blade mode shape, and $\mathbf{\Phi}^{cb}$ are the cantilevered blade modes shapes. The change in the cantilevered blade eigenvalues between the tuned and mistuned blisks $\mathbf{\Lambda}^{\delta,cb}$ is what we define as frequency mistuning. Also, cantilevered blade mode shapes are used in CMM to represent the blade portion of system modes. At lower frequencies, modes tend to be closely spaced in frequency, and the blade motion can be well represented by a single cantilevered blade mode shape. As frequencies increase and veerings become more common, the motion of the blades can be a complicated combination of multiple cantilevered blade mode shapes. Therefore, to obtain accurate predictions, one includes in CMM as many cantilevered blade mode shapes as necessary to predict the blade portion of the system normal mode.

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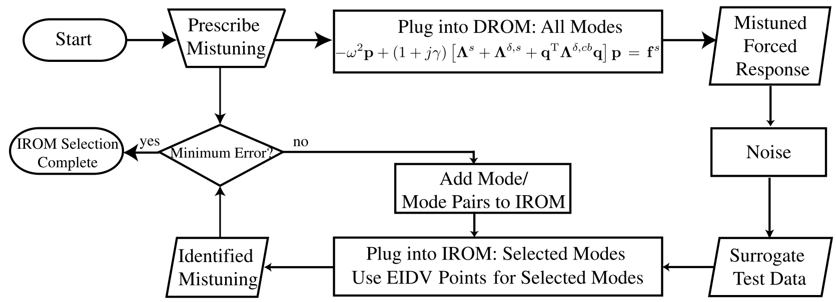


Fig. 1 Flow diagram for IROM evaluation using surrogate data and EIDV measurement iteration.

B. Iterative Measurement Point Selection for Mistuning Identification

Extending the previous work of the authors [20] to higher frequency ranges is nontrivial. For frequency-mistuning identification, the vibration of only a few physical points is measured experimentally. This means that points must be strategically chosen to best represent the physical motion, and thereby the corresponding mode shapes. The increase in complexity of the response of the structure in higher frequency ranges makes it more difficult to identify mistuning. For example, consider a case (often found in the higher frequency ranges) where the topology of the blade motion is highly complicated. This motion could have many nodal lines and antinodes. If measurement points are chosen such that they are on nodal lines, then it would be impossible to identify frequency mistuning, because values of $\mathbf{p}$ would be erroneously measured to be zero. This is an extreme case, but it demonstrates the difficulty of correctly selecting points that are sensitive to frequency mistuning, especially in higher frequency ranges where the blade motion is more complex. Similarly, it also becomes important to optimally choose the measurement points in order to be able to distinguish between mode shapes when only a small number of points per blade are measured in the higher frequency ranges. For example, consider another scenario where a measurement point is chosen at the leading edge tip of the blade. It is possible that several modes have similar displacements at that particular point on the blade, but they are otherwise dissimilar. If only one measurement point is used per blade, then the physical-to-modal transformation in Eq. (1) is ill-conditioned and results in a poor set of modal coordinates to be used in the inverse ROM (IROM) [20]. To address this issue, a novel approach is presented herein, where measurement points are chosen for each IROM in a manner that ensures good conditioning of the modal matrix.

III. Results

A. Iterative Measurement Point Selection for Mistuning Identification

The algorithm for the proposed iterative measurement point selection method is shown in Fig. 1. This approach features two improvements compared with the previous work of the authors [20]. First, for the forward problem, the forced response analysis is used to generate surrogate data for the identification procedure. This takes into account the error that comes from the transformation from physical to modal coordinates. Second, the EIDV measurement points are chosen iteratively as an essential part of identifying frequency mistuning in higher frequency ranges.

The necessity for being selective for the limited amount of measurement points is important to capture the complicated blade motion in higher frequency ranges. To demonstrate this, mistuning identification results are considered for the 24-bladed disk shown in Fig. 2. In Fig. 3a, the frequency-mistuning results are presented for the sixth cantilevered blade mode family using a valid but suboptimal EIDV measurement point (per blade) for the given frequency range, and then they are presented for the optimal point for the IROM selected modes. It is clear that results with lower accuracy are obtained if a suboptimal point is used. Figure 3b shows the mistuning error over all possible IROM sizes. The forced response surrogate data obtained using a direct ROM (DROM) [20] automatically determine that the optimal set of modes happens to be the set of all of the modes in each range.

The ANSYS-generated forced response results for suboptimal EIDV measurement points indicate that a different mode set should be used, and the minimum error would be only slightly less than the error for the optimal point (at that number of modes). However, it is clear that, with the optimal EIDV measurement point in conjunction with the optimal number of modes, the error is minimized with the given set of measurement points. It should also be noted that the DROM surrogate data are obtained using several randomly chosen mistuning patterns to compute the forced response. Then, the algorithm selects the worst error out of those patterns for a given number of IROM modes. This is why the DROM curve has more error than the optimal curve.

B. Multiple Frequency-Mistuning Patterns

To demonstrate the approach, a single physical mistuning pattern is considered for the 24-bladed disk in Fig. 2. Each blade portion is considered to be composed of four segments with differing modulus of elasticities (with variations under 5%). The physical changes result in different frequency-mistuning patterns for each of the six different cantilevered blade modes used in the model.

To demonstrate the current capabilities of the identification algorithm, Figs. 4a–4f show that these six different frequency-mistuning patterns can indeed be identified. It should be noted that the current identification procedure makes use of a combination of the authors' previous work [20] and the measurement point iteration algorithm presented in the previous section. It is clear from Figs. 4a–4f that the frequency mistuning can be well identified for each cantilevered blade mode family. Since the last cantilevered blade mode family has a contribution up to 20 kHz, the results are sufficient to suggest that mistuning can be identified in higher frequency ranges without significant loss of accuracy. As it is well known that forced response amplitudes of mistuned systems are highly sensitive to mistuning, the ability to identify frequency mistuning and, subsequently, model the system using frequency mistuning is critical to obtain accurate predictions. Furthermore, these results

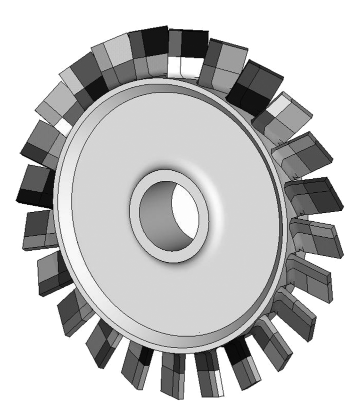
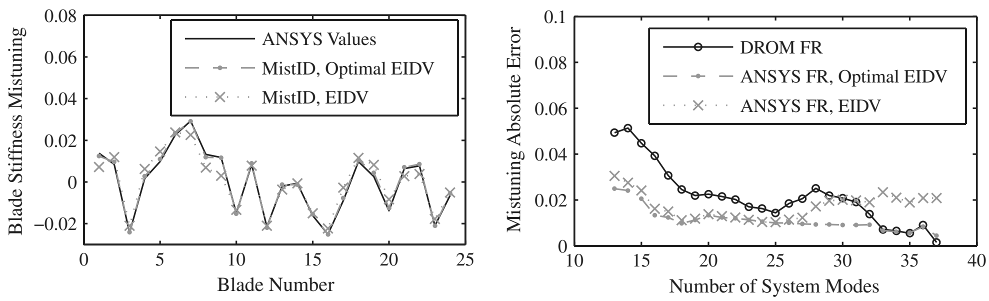
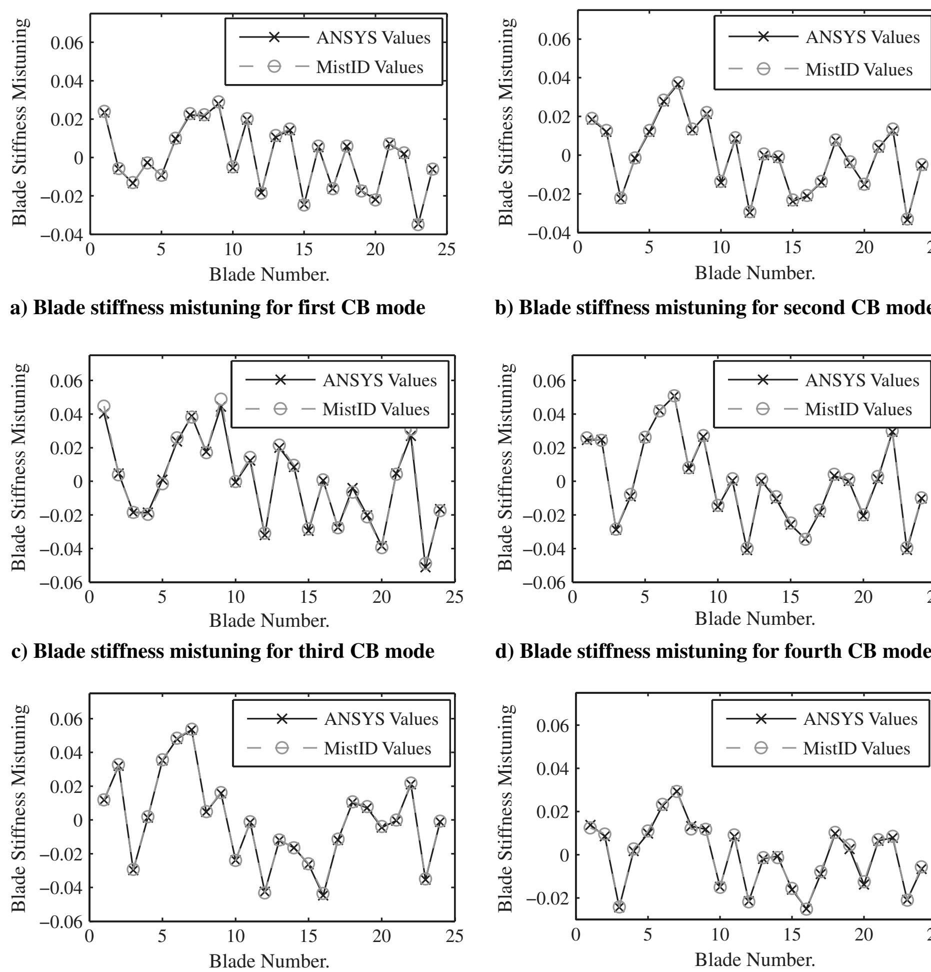


Fig. 2 FEM of a blisk with multiple mistuning patterns.



a) Model updating parameters for sixth CB mode **b) Mistuning error vs mode number for sixth CB mode**

Fig. 3 Justification of iterative EIDV procedure using sixth cantilevered blade mode (FR denotes forced response and CV denotes cantilevered blade). MistID values represent mistuning values obtained using the proposed mistuning identification approach.



a) Blade stiffness mistuning for first CB mode **b) Blade stiffness mistuning for second CB mode**

c) Blade stiffness mistuning for third CB mode **d) Blade stiffness mistuning for fourth CB mode**

e) Blade stiffness mistuning for fifth CB mode **f) Blade stiffness mistuning for sixth CB mode**

Fig. 4 Model updating parameters for first through sixth cantilevered blade modes (0–18,000 Hz). MistID values represent mistuning values obtained using the proposed mistuning identification approach.

demonstrate that six distinct mistuning patterns can be identified, which allows for the modeling of nonproportional mistuning.

IV. Conclusions

Frequency-mistuning identification in higher frequency ranges was discussed. It was noted that, at higher frequencies, there is an increase in the complexity of the motion of the structure that makes systematically choosing sensitive and high responding data points difficult. Therefore, to enable frequency-mistuning identification, a novel measurement point selection algorithm was introduced that enhanced the sensitivity of the measured points to frequency mistuning while rejecting noise. Identification of multiple frequency mistuning in higher frequency ranges was demonstrated. It was also demonstrated that it is possible to identify multiple frequency-mistuning patterns (for a single physical mistuning pattern) using CMM-based identification. The identification of multiple frequency-mistuning patterns allows for more accurate modeling of the dynamics in higher frequency ranges (e.g., using CMM) and makes it possible to model nonproportional mistuning. More details related to this work can be found in Madden et al. [21] Overall, this approach significantly enhances the identification and modeling of mistuning in higher frequency ranges and enables the design and analysis of blisks throughout the entire operating range.

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